CS361, Fall 1995

NFA and Powerset Construction

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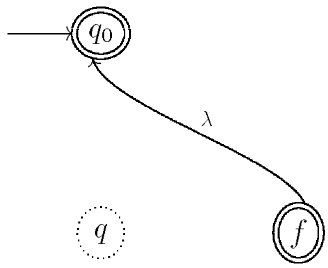
For class given on Sept. 25, 1995, Monday.

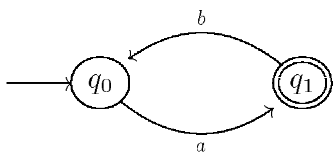
1 NFA (continue Oct. 4th. 1995)

We'd like to show that if $r = L(N)$, then $r^* = L(N')$ for some NFA N'.

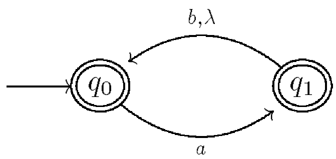
How to construct N'? Is the following construction correct– the idea being making a λ edge from all final states of N to its initial state, which is made a final state if it is not already one?



Sometimes this construction does not work. Consider the following NFA that accept $r = a(ba)^*$

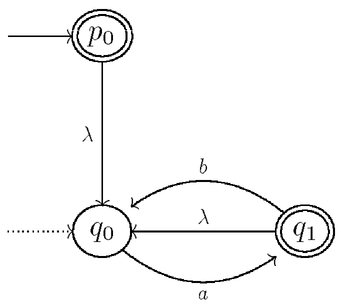


If we follow the above idea, then we would have the NFA below for $r^* = (a(ba)^*)^*$.



However, the string ab is accepted by the NFA but it is not in r^*. Thus, in this case the construction does not work.

Consider the following construction:



We see that this NFA accepts r^*. The general idea is to create a new initial state, make it final, and then make an arc labeled by λ from every final state to the old initial state. This construction can be made formal by specifying the five components of the new NFA in terms of the old NFA.

2 The Powerset Construction

When considering all NFA's as a class, is it $=$, $\subseteq$ or $\supseteq$ to the class of DFA?

Note that for DFA, the transition function has the type

$$\delta : Q \times \Sigma \longrightarrow Q$$

while for NFA the transition function has the type

$$\delta : Q \times \Sigma \longrightarrow \mathsf{Pow}(Q).$$

By regarding a state q as a singleton set $\{q\}$, we see that every DFA is also an NFA. Clearly, NFA's are not necessarily DFA's. However, the powerset construction shows that if $L = L(N)$ for some NFA N, then there exists some DFA M, such that $L = L(M)$. In other words, every NFA can be transformed into an equivalent DFA, "equivalent" in the sense that they accept the same language.

The powerset construction provides a procedure for doing so.

Two notations are used for the construction. One is $\Lambda(A)$, defined to be the set

$$\{p \mid \exists q \in A,\ q \xrightarrow{\lambda}{}^{*}_{M} p\}.$$

So, $\Lambda(A)$ is the collection of all the states that can be reached from some state in A via λ-moves only. Note that $A \subseteq \Lambda(A)$. Intuitively, if $p \in \Lambda(\{q\})$, then "there is a secret passage from q to p."

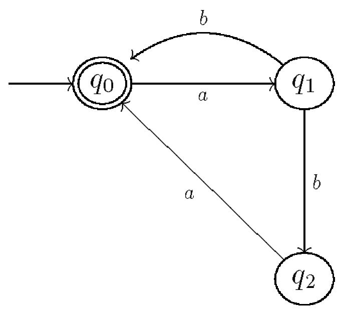
The second notation is to extend the transition function (given by, for example, entries of the form $\delta(p, a) = A$). Define $\delta(B, a) := \bigcup_{p \in B} \delta(p, a)$. This is "all the state you can reach via an a-transition from anywhere in B". Clearly, $\delta(\emptyset, a) = \emptyset$, because "from nowhere we go nowhere". Note that we have carefully used the words "move" and "transition". A move is the one you make when you run an automaton; a transition, instead, is the relation specified in the definition of an automaton.

The Powerset Construction

1. Create a graph G_D with node $\Lambda(\{q_0\})$. Identify this node as the initial state.
2. Repeat the following steps until no more edges are missing.
   (a) Take any node $\{q_i, q_j, \ldots\ldots, q_k\}$ of G_D that has a missing edge for some $a \in \Sigma$.
   (b) Compute $\Lambda(\delta(\{q_i, q_j, \ldots., q_{k\}}, a))$.
   (c) Check if the set from step B is already in G_D. If not, create a new node.
   (d) Make an edge from $\{q_i, q_j, \ldots\ldots, q_k\}$ to $\Lambda(\delta(\{q_i, q_j, \ldots\ldots, q_k\}, a))$ labeled by a.
3. Mark every state of G_D whose nodes contains a final state of the NFA a final state of G_D.

Example. To construct a DFA for $r = (ab + aba)^*$, one can first make an NFA for it, and then convert the NFA to a DFA.

Here is an NFA for $r = (ab + aba)^*$:



According to the powerset construction, an equivalent DFA is: