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Review of Set Notation

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1 Logical Notation

& For example : $(2 < 3)\&(5 < 2)$ is like saying A AND B where A and B are statements. The compound statement is true only when both A and B are both true. Hence, the above statement is false since A is true but B is false.

Or: For example : $(2 < 3)$ Or $(6 > 1)$ is like saying A OR B where A and B are statements. The compund statement is true when either one of the statements A or B is true. It is false when both the statements are false. Hence, the above statement is true.

$\Rightarrow$: This compound statement is false exactly when A is true but B is false. It is denoted by $A \Rightarrow B$. For example: $(5 < 2) \Rightarrow (1 > 2)$ is true.

$\Leftrightarrow$: This operation denotes the equivalence of two statements denoted by $A \Leftrightarrow B$. For example: $(x^2 > 0) \Leftrightarrow (x > 0)$ which is false.

$\exists$ - EXISTENTIAL QUANTIFIER: There is some x such that $p(x)$ holds. $\exists x, x^2 = 1$.

$\forall$ - UNIVERSAL QUANTIFIER: For all x, $P(x)$ holds. $\forall x, x^2 > 0$.

There are two ways to specify a set. For a finite set, list all the members of the set. For example, $\{a, b, c, d, e\}$. $\{\ \}$ denote a set. Items within the $\{\ \}$ are members of the set. For a general set, $\{x \mid P(x)\}$ is a set made up of all elements x with property P. For example, $\{x \mid \exists k, 2k = x\}$.

$x \in A$ means that x is a member of A. $\emptyset$ denotes the empty set. There are no elements in the set. $\{\emptyset\} = \emptyset$ is a false statement. Think of the empty set as a basket and a set as a basket. If a set is composed of the empty set, you have a basket within a basket, so $\{\emptyset\}$ is not the empty set.

SET EQUALITY AND CONTAINMENT: $A \subseteq B \Leftrightarrow \forall x (x \in A \Leftrightarrow x \in B)$. This means $A = B \Leftrightarrow (A \subseteq B)\ \&\ (B \subseteq A)$. For example, we have $\{a,b\} = \{b,a\}$. $\{a,b,a\} = \{a,b\}$.

SET OPERATIONS

UNION: $\{a,b\} \cup \{1,2\} = \{a,b,1,2\}$.

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

INTERSECTION: $\{a,b\} \cap \{1,2\} = \emptyset$. $A \cap B = \{x \mid x \in A\ \&\ x \in B\}$.

DIFFERENCE: $\{a,b\} - \{1,2\} = \{a,b\}$. $A - B = \{x \mid x \in A\ \&\ x \notin B\}$.

SET CONSTRUCTIONS

PRODUCT: $A \times B = \{(a,b) \mid a \in A\ \&\ b \in B\}$. Note that $A \times B \neq B \times A$. Example: $\{(2,1)\} \neq \{(1,2)\}$.

POWERSET: $\mathsf{Pow}(A) = \{X \mid X \subseteq A\}$. Example: $\mathsf{Pow}(\emptyset) = \{\emptyset\}$.

$\mathsf{Pow}(\{a,b,\{a,b\}\}) = \{\emptyset, \{a\}, \{b\},$
$\{\{a,b\}\}, \{a,b\}, \{a,\{a,b\}\},$
$\{b,\{a,b\}\}, \{a,b,\{a,b\}\}\}$.

For a set X, we denote by $\mid X \mid$ the number of elements in X. We have $\mid \mathsf{Pow}(A) \mid = 2^{|A|}$.

RELATIONS AND FUNCTIONS: $R: A \to B$ is a relation if and only if $R \subseteq A \times B$. $f: A \to B$ is a function if $f \subseteq A \times B$ and every element of A is mapped to one and only one element of B.

EQUIVALENCE RELATIONS: $R \subseteq A \times A$ is an equivalence relation if it has the following three properties:

1. It is reflexive.

$$\forall x[x \in A \Rightarrow (x, x) \in R].$$

2. It is transitive.

$$(x, y) \in R \;\&\; (y, z) \in R \Rightarrow (x, z) \in R.$$

3. It is symmetric.

$$(x, y) \in R \Rightarrow (y, x) \in R.$$

To check our understanding of the concept, we can try to see if the rlation $\emptyset$ on a set A is an equivalent relation.

The properties required of an equivalence relation are just a generalization of the equality relation on integers, for which we have $a = a$, if $a = b$ then $b = a$. if $a = b$ and $b = c$ then $a = c$.

There are two ways to express that a pair (a, b) is in a relation R. One is to write $a\,R\,b$, and the other is $(a, b) \in R$.

In terms of the notation here, we could write, for the equality relation on integers, $(a, a) \in =$, if $(a, b) \in =$ then $(b, a) \in =$, and if $(a, b) \in =$ and $(b, c) \in =$ then $(a, c) \in =$.