CS361, Fall 1995

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Lecture Notes of 10/04/95

Topic: NFA

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1 Introduction

Recall that a DFA is a tuple

$$M = (Q, \Sigma, q_0, \delta, F),$$

where $\delta : Q \times \Sigma \to Q$ is a function.

2 Definition of NFA

2.1 Definition 2.4

A **nondeterministic finite acceptor** or **NFA** is defined by the quintuple

$$M = (Q, \Sigma, q_0, \delta, F),$$

where Q, Σ, q_0, F are the same as the ones for DFA, but the transition funtion is different from that of a DFA:

$$\delta : Q \times (\Sigma \cup \{\lambda\}) \to \mathsf{Pow}(Q),$$

which is a function from $Q \times (\Sigma \cup \{\lambda\})$ to the power set of Q.

2.2 The defferences between NFA and DFA

The difference between DFA and NFA is the transition function. First, in NFA, the range of δ is the powerset $\mathsf{Pow}(Q)$, so that its value is not a single element of Q, but a subset of Q. This subset defines the set of all possible states that can be reached by the transition function. Second, we allow λ as the second argument of δ. This means that an NFA can make a transition without consuming an input symbol.

2.3 What is a string accepted by an NFA?

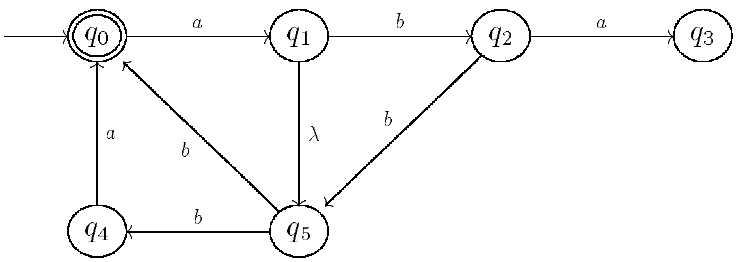
A string is accepted by an NFA if there exists a sequence of possible moves that will put the machine in a final state when all the symbols in the input string are used. In other words, **a string is accepted by an NFA if and only if you can find a succesful transition for it to a final state**. Using set notation, we have

$$L(M) = \{a_1 a_2 \cdots a_n \quad | \ \exists q_0, s_1, s_2, \ldots s_n$$
$$s_1 \in \delta(q_0, a_1) \ \& \ s_2 \in \delta(s_1, a_2) \ \& \ \cdots s_n \in \delta(s_{n-1}, a_n) \ \& \ s_n \in F\}.$$

Example 1 Which of the following strings

$$(1).aa \ \ (2).aba \ \ (3).abb \ \ (4).ab \ \ (5).abab$$

are accepted by the following NFA?

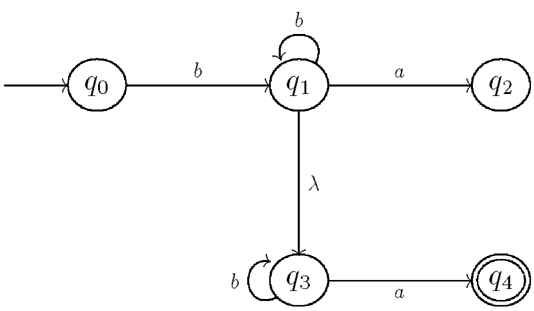


ANSWER: Only $(2).aba, (4).ab, (5).abab$ are accepted.

Example 2 Which of the following strings

$$(1).ba \ \ (2).ab \ \ (3).bb \ \ (4).b \ \ (5).bba$$

are accepted by the following NFA?



ANSWER: Only $(1).ba \ \ (5).bba$ are accepted.

3 Theorem

With respect to NFA, we have

Every regular language is accepted by some NFA.

Recall that a language is called regular if it can be described by a regular expression.

The kep step for the proof is to show that if $r_1 = L(N_1)$ and $r_2 = L(N_2)$, then there are NFA's N_3, N_4, N_5 such that

$$r_1 + r_2 = L(N_3),$$
$$r_1 r_2 = L(N_4),$$
$$r_1^* = L(N_5).$$

4 The extended transition function

For an NFA, the extended transition function is defined so that $\delta^*(q_i, w)$ contains q_j if and only if there is a walk in the transition graph from q_i to q_j labeled by w. This holds for all q_i, q_j in Q and w in Σ^*.

With the notation for extended transition function, we can rewrite the language an NFA

$$M = (Q, \Sigma, \delta, q_0, F)$$

as

$$L(M) = \{w \mid w \in \Sigma^* \ \& \ \delta^*(q_0, w) \cap F \neq \emptyset\}.$$

In words, the language contains all strings w for which **there is** a walk labeled w from the initial vertex of the transition graph to **some** final vertex.