SUMMARY OF RESEARCH

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My research over the recent years has been focused mainly on two areas: one is *domain theory* as used in the semantics of programming languages, and the other is the application of domain theoretic tools for *partial logics and nonmonotonic reasoning.* Note that this is not a summary of all the recent research.

1 Domain Theory

The main effort has been on two kinds of topics in the area. One is dI-domains and stable functions, and the other is representing various categories of domains in terms of information systems, in the sense of Scott. Of course, some of the work is a combination of the two. Here is an itemized description of the work.

Stable Domains. The category of dI-domains and stable functions is a relatively intricate, yet elegant framework for semantics of programming languages. This category was discovered by Berry as a denotational model of a typed lambda-calculus. Since then, it has found applications in linear logic, concurrency, polymorphism, sequential computations, and object-based semantics for interference-free imperative programs.

- The internal structure of a stable function is captured by its trace, representing necessary and sufficient information which determines the input-output behavior of a stable function. An axiomatization of traces is provided in *dI-domains as prime information systems* (Information and Computation, vol 100, 1992, pp.151-177; An earlier version of the paper was presented at ICALP 89'). The axiomatization makes it much easier to work with stable functions. It is an effective technical tool used in related research.
- dI-domains can be represented by event structures in the sense of Winskel. Event structures, on the other hand, have been used as a model for concurrency, partly because they are closely related to Petri nets. This provided inspiration to work on finding a monoidal closed category of event structures, with the ultimate goal of providing a link between Girard's linear logic and concurrent computation. Results on this topic appeared in *Some monoidal closed categories of stable domains and event structures* (*Mathematical Structures in Computer Science*, vol 3, 1993, pp. 259-276. An earlier version of the paper was presented at MFPS 91').

- Just as *full abstraction* is concerned with the relationship between operational and denotational semantics, *domain logics* establish a bridge between denotational and axiomatic semantics. In a framework of domain logic, one uses the paradigm of Abramsky and Smyth which provides a correspondence between observable computations and topological spaces. In this paradigm, programs correspond to points, properties of programs correspond to open sets, and types correspond to topological spaces. In *Logic of Domains* (Birkhauser, Boston, 1991), the expressive power of domain logics has been studied, through the extension to a mu-calculus. Logics for stable domains and SFP domains have been investigated, with an emphasis on taking information systems as the backbone. Soundness and completeness of the logics have been achieved.

- Note that when it comes to dI-domains Scott open sets do not capture stable functions. A new "topological" framework is needed, with a disjunctive feature. This is reported in detail in *Stable neighborhoods* (*Theoretical Computer Science*, vol 93, 1992, pp. 143-157).

- There are two interesting maximality questions related to stable functions. One is internal maximality and the other is external maximality. The internal maximality is about characterizing stable functions which preserve maximal elements. This is motivated by the intuition that maximal, isolated elements should denote terminating processes, as well as by various notions of totality, such as totality using games and totality spaces. The results of the paper, *Maximal stable functions* (*Theoretical Computer Science*, vol 146, 1995, to appear), are: (i) a characterization of stable domains for which trace-maximum is equivalent to preserving maximal elements, and (ii) a topological characterization of maximal stable functions.

- External maximality, on the other hand, deals with the existence of various categories with certain properties, such as cartesian closure. A recent paper, entitled *The largest cartesian closed category of stable domains* (submitted to *Theoretical Computer Science*), shows that the category of dI-domains is the largest cartesian closed category within Scott domains, with exponential being the stable function space. This settles the question of exactly what kind of roles Berry's axioms (d and I) play.

Information Systems. Scott's idea of an information system plays at least three important roles: pedagogical, technical, and foundational. The pedagogical role is what motivated Scott's original paper – making domain theory more accessible. The technical role can be seen as a direct consequence of the former. By representing domains as information systems, one can work with more concrete and familiar notions of sets, relations, and containment. This is specially true for solving domain equations via information systems, as well as for determining the existence of universal objects through the use of Droste and Gobel's amalgamation condition. The third role of information systems has been used in two ways in my work, one by taking it as the backbone of domain logics, and the other by taking it as the starting point of the work on properties of the nonmonotonic consequence relation (see the next section).

- The paper entitled *dI-domains as prime information systems* (*Information and Computation*, vol 100, 1992, pp.151-177) provides a representation of the category of dI-domains as information systems. Constructions on these systems are introduced and shown to be continuous in the big cpo of prime information systems. This serves as a theoretical foundation for recursively defined dI-domains.
- Plotkin's category SFP is a domain theoretic framework in which to give meanings to parallel imperative programming languages. SFP domains can also be represented by information systems (in a more general sense). This is reported in *A representation of* SFP (*Information and Computation*, vol 110, no. 1, pp. 233-263, 1994). Constructions on these systems are introduced and shown to be continuous, establishing a theoretical foundation for recursively defined SFP domains.
- In *Disjunctive systems and L-domains* (ICALP 92', *Lecture Notes in Computer Science* 623,1992, pp.284-295), Jung's L-domains have been represented as information systems of a disjunctive nature.
- It proves effective to study quasi-prime algebraic domains via the technique of information systems, showing that they can be made cartesian closed and monoidal closed (*Quasi-prime algebraic domains*, submitted to *Theoretical Computer Science*). The existence of a saturated quasi-prime algebraic domain has been established as well (*Universal quasi-prime algebraic domains*, MFPS 93', *Lecture Notes in Computer Science* 802, pp. 454-473).

2 Nonmonotonic Reasoning

Nonmonotonic reasoning refers to the kind of reasoning people use in everyday life, so that they can jump to conclusions and exhibit common sense. The very nature of nonmonotonicity sounds anti-logical (and it is). However, people use nonmonotonic reasoning all the time. It can be taken a step further: because of the inherent inability to achieve complete information (with time constraint), human beings cannot function in the real world without some sort of nonmonotonic reasoning mechanism.

The key idea of *default domain theory*, an area initiated by William Rounds and myself, is that the nonmonotonic feature of nonmonotonic reasoning should reside in the semantic models, and not in the axioms and inference rules of the logic. Because nonmonotonic reasoning is fundamentally about partial information, information systems become an appropriate and powerful tool in studying basic aspects of nonmonotonic reasoning.

- One of the most influential works in nonmonotonic reasoning has been Reiter's default logic, or, more precisely, the concept of an *extension*. However, extensions are in some cases not robust enough. In a joint paper with Rounds, entitled *Domain theory meets default logic* (*Journal of Logic and Computation*, vol 5, no. 1, pp. 1-25, 1995), the notion of *dilations* is introduced, based on the information systems setting. It has been shown that, among other things, dilations exist for all *rational* defaults and, extensions, when they exist, are minimal dilations.

- The basic properties any logical consequence relation should have are essentially coded in an information system. What kind of properties should a nonmonotonic consequence relation have? What should a nonmonotonic consequence relation mean in the first place? These issues are addressed in the paper *Nonmonotonic consequences in default domain theory* (with Rounds, *Annals of Mathematics and Artificial Intelligence*, to appear). The main result of the paper is a representation theorem which concludes that there is a one-to-one correspondence between *default information systems* and nonmonotonic consequence relations. This theorem dictates the basic laws of a nonmonotonic consequence relation. The commonly accepted law of *Cautious Monotony* (due to Gabbay and others) has been found to be questionable in default domain theory.

- Domain theoretical and topological aspects of the nonmonotonic consequence relation have been studied in *Defaults in domain theory* (with Rounds), which was presented at MFPS 94' and has been submitted to *Theoretical Computer Science*.

- In a couple of other papers (joint with Rounds), various kinds of logics with default semantics have been investigated. The logics are all partial, or three-valued. One of the characteristics of the approach is that it is default semantics based. This means semantics comes first, axioms and rules second. So far we have studied a partial first order logic (*Logical considerations of default semantics*, *Annals of Mathematics and Artificial Intelligence*, to appear; *Attunement to constraints in nonmonotonic reasoning*, *Proc. of the 4th Conference on Information-Oriented Approaches to Logic, Languages, and Computation*) and a partial modal logic (*Nonmonotonic reasoning in a monotonic modal logic*, manuscript).

Default domain theory shares common ideas with Barwise's situation theory. As future work, we will be investigating the relationship between the two approaches. We are also interested in applications of default domain theory in areas such as natural language processing, logic programming, and programming languages.